Financial Networks and the Propagation of Systemic Risk*

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Abstract

We illustrate how a network-based analysis can be useful to the evaluation of systemic risk, highlighting the abilities of a network model in terms of identification and measurement of the system-wide effects. Beginning with the methodological framework used in the social interactions literature, we discuss the use of behavior-based models in the financial markets context and relate our approach to that used in the epidemiological literature. Using these ideas, we define a new measure of systemic risk. Our measure will differ from existing approach in that it will depends on the specific network architecture and will be a function of the strategic behavior of agents in the system. The measure is a quantification of the average impact of a shock that emerges as the result of the strategic reaction of market participants. We provide an application of this approach discussing the role of correlated trading strategies in fully electronic exchanges. While such markets offer no ability for traders to choose their transaction partners, the realized pattern of trades resembles a highly organized network. Importantly, these network patterns are closely related to profitability in the market; certain positions in the network are more valuable than others. As well, the observed structure of the network implies a very large impact of shocks to the system. We conclude with some policy implications and suggestions for future research.

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1 Financial networks and systemic risk

The growth in interest in network spillovers in finance stems from an increased recognition that economic theory has difficulties in explaining a number of economic phenomena without acknowledging the importance of interdependence between preferences, constraints, and expectations.

With the financial crisis and increasing concerns about financial integration and stability as a leading example, a large number of theoretical papers have begun to exploit the network of mutual exposures among institutions to explain financial contagion and spillovers. Allen and Babus (2009) survey the growing literature. From an empirical point of view, there is little guidance in the literature on how to estimate the propagation of financial distress. Approaches for estimation of network influences is widely varied in the financial literature. Some use instrumental variables (Leary and Roberts, 2010), some use summary statistics of network characteristics (Ahern and Harford, 2010, Blocher, 2011, Hoberg and Phillips, 2010, 2011, Hochberg et al., 2007, Lin et al. 2011), others use the tools of random networks (Allen and Gale, 2000, Amini et al. 2011, Brunnermeier and Pedersen, 2009, Gai et al, 2011).

Theoretically, a number of mechanism have been proposed that can generate systemic risk. One comes from Herring and Wachter (2001), in which agents are simultaneously impacted by a shock to underlying asset prices. While not a network approach per se, this paper typifies a large body of research which looks at common bank incentives in the face of a shock. The second mechanism is the Allen and Gale (2000) or Freixas et al (2000) mechanism in which, given a network generated at random, the default of a given entity can lead to domino-like series of subsequent defaults based on exposures to the defaulting entity. A newer class of models updates the network approach to specify that links between banks are based on preferential attachment (Jackson and Wolinsky, 1996); that is, while links are still random, banks may be more likely to link with another of similar type. For example, Allen, et al. (2011) illustrate using this approach how the accumulation of exposure to shocks depends on the incentives for individual banks to diversify holdings.

None of above approaches, either theoretical or empirical, however, is able to capture the precise *cascade in behavior* that occurs between interconnected agents that occurs following an idiosyncratic shock.[1] Following this idea, in this chapter we describe to what extent a network model of behavior can be useful in the evaluation of systemic risk, highlighting the abilities of a network-based analysis in terms of identification and measurement of system wide effects.

We define systemic risk as the degree to which minor changes in the actions or outcomes of a single entity can cascade into a system-wide effect. Our particular view of systemic risk emphasizes the role of behavior and its impact on equilibrium outcomes. When a single agent changes, others do as well as a result of the strategic interaction between them. Thus, in order to understand systemic risk in a given market, we have to understand how a change in a single agent changes the equilibrium ex-post. This can be understood as a cascade in behavior. When a single event occurs, the corresponding market reaction will be based both on the information conveyed by the transaction itself, as well as the cascading impact of agents adjusting their behavior to reflect the new market environment. As an empirical example, Kirilenko et al (2011) provides detailed descriptions of how, during the Flash Crash of May 6, 2010, the triggering event of a large sell order led to a range of actions by different traders in the market. Indeed, they find clearly defined groups of traders who behaved in markedly similar ways during this period of stress. This change in behavior as a result of the new market environment is captured in some random networks models,

[1] Kirilenko at al. (2011) assess the role of networks in propagating changes in returns across traders in fully electronic exchange markets.

notably Brunnermeier and Pedersen (2009), but to date, to our knowledge, is only fully modeled in complex networks in Cohen-Cole et al. (2011).

Our measure of systemic risk is a quantification of the behavioral response of agents to changes in the system. This measure of risk is significantly different than others proposed in the literature in two ways. One, it is derived using the structural properties of the entire network, i.e. all the actual (direct and indirect) connections present in the network. Two, it emphasizes the cascades that occur as a result of changes in behavior as propagated through the network. Significantly, it suggests that large impacts can occur in the absence of defaults. The flash crash of May 6, 2010 is a key example.

Thus, one can think of the 'network' as a tool to describe the outcome of the behavior of agents in this market. Because of the complexity of interactions, describing the risk present in the marketplace brings many challenges to traditional, non-network based, regression methods. The potential gain from a network approach is the ability to use methodologies that capitalize on the network structure itself to measure the properties of the system.

Specific models of diffusion processes on complex networks in epidemiology (e.g. Pastor-Satorras and Vespignani, 2001; Durrett, 2010) emphasize the relationship between the structural properties of the networks and the dynamics of the processes defined on the networks. These kind of studies are particular useful to define suitable procedures to stop the propagation of epidemics (Moreno et al., 2003; Badham and Stocker, 2010; Kiss et al., 2008), the corollary of which in financial crisis is obvious. Even though the channels of propagation of financial distress are different from those of medical diseases, those models may be helpful to understand the dynamics of the financial system, as well as to devise efficient and fast actions for the protection of financial networks against shocks.

As an example, the largest eigenvalue of the adjacency matrix; that is, the matrix that represents the connections in the network, is related to the epidemic threshold of a network, which is the critical ratio between the propagation rate and the recovery rate of a disease above which epidemics ensue. Indeed, a way to see this connection is to note the connection between the threshold in epidemiological models and the Nash equilibrium in economic ones. The connection between these is intellectually compelling; the Nash equilibrium in economic models reflects the best response of strategic agents in a market. In a network setting, it reflects the best response conditional on the network connections. The best response conditional on network structure has been shown under some mild conditions in Ballaster et al. (2006) to be proportional to a measure which is bounded above by the inverse of the largest eigenvalue of the network. We return to this finding below, but note here that the same value is found in epidemiological models as the 'threshold' tipping point for an epidemic (Wang et al., 2003; Chakrabarti et. al. 2008).

Our approach to the analysis of systemic risk borrows from these various (seemingly unrelated) strands of the literature. An important part of the attractiveness of this approach is the fact that, although it is highly technical in principle, its practical implementation is straightforward. After the network of connections has been mapped, the approach reduces to the specification of an enriched linear regression model that can be estimated using standard software via maximum likelihood.

This chapter is organized as follows. We start in Section 2 by discussing the approach of the social interactions literature to modeling interconnections. The social interactions literature is particularly instructive here as it has fully developed models of behavior and equilibria. The notion of equilibrium here will help to illustrate that small shocks can cascade through the market based solely on the change in strategic behavior; that is, one can generate systemic risk without defaults. Section 2 continues by explaining the main identification challenges of this approach and to what extent those issues can be tackled using a network approach. Section 3 contains an application to futures markets. Section 4 concludes.

2 Diffusion-like processes over networks

2.1 Behavioral foundation

We consider a behavioral foundation from the social interaction literature. Consider a set of financial market agents (banks, traders, etc) that each attempt to maximize returns. We use the Blume et al (2010) baseline model as a way to study the joint behavior of agents which are members of some group k. We want to be able to describe, in a probability sense, choices that agents make. Denote the choices of agent i in group k as $\omega_{i,k}$. To be specific, we define that choices are made from elements of some set of possible choices, $\Omega_{i,k}$. This set is both individual- and group-specific. For every individual i, we must track the choices of other agents in $i's$ group. We denote these as $\omega_{-i,k}$. That is, the trading choices made by trader i depend on the trading choices made by agents $-i$. We define five influences on trader behavior. Each of these have different implications for modeling:

- x_i – an N-length vector of observable agent-specific characteristics (big traders, small traders, etc)
- y_k – an V-length vector of observable group-specific characteristics (frequency of trades, etc.)
- $\mu_i^e(\omega_{-i,k})$ – A unobservable probability measure that describes the beliefs agent i possesses about behaviors of others in the group

- ε_i – A vector of random unobservable agent-specific characteristics associated with i, unobservable to the modeler, and

- α_k – A vector of unobservable random group-specific characteristics

The object $\mu_i^e(\omega_{-i,k})$ is a useful one as it describes a wide variety of cases in the literature. This includes the cases where agents know the actions or intended actions of others in their group. This amounts to placing a probability of 1 on one element of the set of possible choices.

Individual choices are now represented as the maximization of some payoff function:

$$\omega_{i,k} = \arg\max_{\lambda \in \Omega_{i,g}} V\left(\lambda, x_i, y_k, \mu_i^e(\omega_{-i,k}), \varepsilon_i, \alpha_k\right)$$

Notice that this is a decision that is based on preferences, beliefs, and constraints. Preferences are in the V functional form, beliefs about the actions of others are contained in the variable $\mu_i^e(\omega_{-i,k})$ and constraints are in the set of possible actions, $\Omega_{i,k}$. To close this model, one needs some assumption on how to determine the beliefs of agents. We follow the literature and require self-consistency (equivalent to rational expectations).[2] That is, in equilibrium, subjective beliefs, $\mu_i^e(\omega_{-i,k})$, match the objective conditional probabilities of the behaviors of others given i 's information set F_i :

$$\mu_i^e(\omega_{-i,k}) = \mu(\omega_{-i,g}|F_i)$$

This point is crucial. It states that the beliefs one conditions on in making choices will match actual choices. This ensures that as one individual changes behavior, those changes are propagated throughout the population along the relationships between individuals.

[2] Blume et al (2010) discuss that the equilibrium in this model can thus be seen as a Bayes-Nash equilibrium of a simultaneous-move incomplete information game.

Blume et al (2010) discuss that the distinction between y_k and $\mu_i^e(\omega_{-i,k})$ is of particular importance in the econometrics literature. Following Manski (1993), the former is known as a contextual effect whereas the latter (including the case of perfect foresight) is known as an endogenous effect. The importance of this distinction is that contextual interactions involve the interactions of predetermined (from the perspective of the model) attributes of one agent affecting another whereas endogenous interactions allow for the possibility of simultaneity of interactions in individual outcomes. This distinction leads to an identification problem that we discuss at length in the next section. In short, a reduced form regression of choices on the average behavior of others leads to identification failure as the estimates cannot account for the fact that the data only reached its observed point as the result of a Nash equilibrium process. A reduced form assumes, incorrectly, instead that there is a exogenous generative process that governs the relationships between individuals. It also illustrates tangibly that the observed data are an equilibrium outcome.

As an example of this distinction: heterogeneity in trader returns depends on private information. Larger firms tend to have greater access to information. If size is a sufficient statistic for information, then the mechanism for higher returns is observable and thus constitutes an element of the vector x_i. One wishes instead to understand how the correlation in behavior, independent of what could be gleaned from size similarities, generates correlations in returns. This is an example of a determinant of individual trader outcomes that can produce a relationship between individual and network characteristics. This is true even when the characteristic is purely individual. The identification issue below is whether the different explanations are distinguishable given different types of data.

2.2 Empirical model and identification issues

From the foundation in the above section, we can outline its empirical counterpart as a linear-in-means model. Assume that there are N agents divided in $k = 1, ..., K$ groups, each with n_k members, $i = 1, ..., n_k$, $\sum_{k=1}^{K} n_k = N$. The interaction scheme can be represented by a matrix $G = \{g_{ij}\}$ whose generic element g_{ij} would be 1 if i is connected to j (i.e. interacts with j) and 0 otherwise. For example, consider 10 traders, 5 from each of two groups: $k = 1, 2$. Traders $i = \{1, 2, 3, 4, 5\}$ belong to group $k = 1$ and traders $i = \{6, 7, 8, 9, 10\}$ belong to group $k = 2$. This means each groups trades within its own ranks but not outside it. The associated G matrix is

$$G = \begin{array}{cc} & \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & & & & & \\ 1 & 1 & 1 & 1 & 1 & & & & & \\ 1 & 1 & 1 & 1 & 1 & & & & & \\ 1 & 1 & 1 & 1 & 1 & & & & & \\ 1 & 1 & 1 & 1 & 1 & & & & & \\ & & & & & 1 & 1 & 1 & 1 & 1 \\ & & & & & 1 & 1 & 1 & 1 & 1 \\ & & & & & 1 & 1 & 1 & 1 & 1 \\ & & & & & 1 & 1 & 1 & 1 & 1 \\ & & & & & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \tag{1}$$

The $i-th$ row of the matrix G, denoted by G_i, indicates the group of trader i. In the matrix above, trader 5 has traders 1,2,3, and 4 in its group.

The linear-in-means model can be written as:

$$\omega_i = \alpha + \beta E(\omega|G_i) + \delta E(x|G_i) + \gamma x_i + \varepsilon_i, \qquad E(\varepsilon_i|G_i, x_i) = 0 \tag{2}$$

where $E(\omega|G_i)$ contains the averages of the ω's in the group of agent i, and captures the fact that the agent i makes her/his decisions based on the expected behavior of the agents in her/his group. Similarly, $E(x|G_i)$ contains the averages of the x's in the group of individual i. It is meant to capture the effects of the shared characteristics of the group, which are typically assumed to be the group level averages of the individual characteristics.[3]

The core problem that emerges in estimation with this type of model is Manski's reflection problem.

We saw in our behavioral discussion that the typical assumption one makes to close the model is that the expected behavior of group members corresponds in equilibrium to the actual average behavior of the group members. It is thus easy to show, by simply averaging model (2) over group G_i, that $E(\omega|G_i)$ is a linear combination of the other regressors:

$$E(\omega|G_i) = \alpha + \beta E(\omega|G_i) + \delta E(x|G_i) + \gamma E(x|G_i) \tag{3}$$

$$E(\omega|G_i) = \frac{\alpha}{1-\beta} + \frac{\delta+\gamma}{1-\beta} E(x|G_i) \tag{4}$$

The fact that $E(\omega|G_i)$ is a linear combination of the other regressors implies that we cannot identify if a trader's action is the cause or the effect of peers' influence, i.e. the endogenous effects (captured by the parameter β) cannot be distinguished from exogenous effects (captured by the parameter δ) (Manski, 1993).

Formally, plugging equation (4) in (3) yields the reduced form regression:

$$\begin{aligned}
\omega_i &= \alpha + \beta \left[\frac{\alpha}{1-\beta} + \frac{\delta+\gamma}{1-\beta} E(x|G_i) \right] + \delta E(x|G_i) + \gamma x_i + \varepsilon_i \\
\omega_i &= \frac{\alpha}{1-\beta} + \frac{\beta\delta+\beta\gamma}{1-\beta} E(x|G_i) + \delta E(x|G_i) + \gamma x_i + \varepsilon_i \\
\omega_i &= \frac{\alpha}{1-\beta} + \frac{\beta\gamma+\delta}{1-\beta} E(x|G_i) + \gamma x_i + \varepsilon_i \\
\omega_i &= \alpha^* + \delta^* E(x|G_i) + \gamma^* x_i + \varepsilon_i \qquad (5)
\end{aligned}$$

[3] It should be apparent that the matrix G provides a simple way to generate the averages over the members of the group. When the matrix G is row-normalized, i.e. each row sums to one, one needs only to matrix multiply the ω and x vectors by such a row normalized G to get, for instance, $E(\omega|G_i)$.

where $\alpha^*, \delta^*, \gamma^*$ are super-parameters equal to:

$$\alpha^* = \frac{\alpha}{1-\beta}, \gamma^* = \gamma, \delta^* = \frac{\beta\gamma + \delta}{1-\beta}$$

It is now apparent that equation (5) contains only 3 coefficients, one less than the structural model has in (2). The true parameters cannot be recovered from this system of equations:

$$\begin{aligned} \alpha &= (1-\beta)\,\alpha^* \\ \gamma &= \gamma^* \\ \delta &= \delta^*\,(1-\beta) - \beta\gamma \end{aligned}$$

Identification thus fails.

The basic difference of this model when it is applied to network data lies in the different structural properties of the matrix G, which provide the key for identification. In real-world networks, an individual is not influenced by all others in a given group, which means an higher heterogeneity in links in the associated matrix. Going back to our previous example with 10 traders, an illustrative network can be represented by:[4]

$$G = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & & & & & \\ 0 & 0 & 0 & 1 & 1 & & & & & \\ 0 & 1 & 0 & 0 & 0 & & & & & \\ 0 & 0 & 1 & 0 & 1 & & & & & \\ 1 & 0 & 0 & 0 & 0 & & & & & \\ & & & & & 0 & 1 & 1 & 0 & 1 \\ & & & & & 1 & 0 & 1 & 1 & 1 \\ & & & & & 1 & 1 & 0 & 1 & 1 \\ & & & & & 0 & 1 & 1 & 0 & 1 \\ & & & & & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \tag{6}$$

[4] We set the diagonal elements equal to 0 by convention.

The 1's in the matrix indicate, for example, that trader 1 interacts with traders $2, 3, 4, 5$, trader 2 with $4, 5$, trader 3 with 2, etc. Note as well, that trader 6 does not interact with trader 9.

Configuration (1) corresponds to a complete network scheme whereas in (6) the network is incomplete. In a complete network scheme, G is block-diagonal with each block populated by ones. The consequence of this interaction scheme is that network connection are fixed across agents: if i and j have the same connections, then the two groups coincide, i.e. $G_i = G_j$. In the second case (incomplete network) G is not block-diagonal, implying that connections are agents specific: if i and j do not have the same connections, then j's connections G_j does not coincide with G_i. Indeed, if the network is incomplete, agents are not affected by all agents, but only by the indicated connections.

Under this structure, if we take the expectation of ω_i conditional on each agent's group (as in equation 3), we get:

$$E(\omega|G_i) = \alpha + \beta E\left[E(\omega|G_j)|G_i\right] + \delta E\left[E(x|G_j)|G_i\right] + \gamma E(x|G_i)$$

It is now apparent that this equation contains as many parameters as the structural model (2). Identification is achieved. That is, network effects are identified if we can find two agents in the economy that differ in the average connectivity of their direct contacts. The precedent for this is Cohen-Cole (2006), a proof of identification that includes multiple groups. The mathematics of the proof for incomplete networks is available in Bramoullé et al (2009), and applications are in Calvo-Armengol et al. (2009) and Cohen-Cole et al. (2010). Of course, networks in financial markets have a very rich structure and identification essentially never fails. Thus, using the architecture of networks we can always obtain estimates of the relevant structural parameters.

Notice as well that the presence of multiple networks, K, in the economy either at a point in time or over time allows the use of fixed-effect estimation. Fixed effect are useful here because that also allow one to control for unobserved heterogeneity. In other words, having network data one can exploit two sources of variation: between networks and within networks (i.e. across individuals in a given network). Similarly to a panel data context where we observe individuals over time, we can thus estimate a fixed-effects model where the effects are fixed across individuals in the same networks (instead of fixed across time).

2.3 Interpretation: the systemic risk multiplier

For ease of interpretation, we write the model (2) in matrix notation:

$$\boldsymbol{\omega} = \beta G\boldsymbol{\omega} + \gamma\mathbf{x} + \delta G\mathbf{x} + \boldsymbol{\epsilon}, \qquad E\left[\boldsymbol{\epsilon}|\mathbf{x}\right] = 0 \tag{7}$$

where $\boldsymbol{\omega}$ is a $N \times 1$ vector of outcomes of N agents, $\mathbf{x}$ is a $N \times V$ matrix of V variables that may influence agent behavior but are not related to networks, G is the row standardized $N \times N$ adjacency matrix from above that formalizes the network structure of the agents, $\boldsymbol{\iota}$ is a $N \times 1$ vector of ones and $\boldsymbol{\epsilon}$ is a $N \times 1$ vector of error terms, which are uncorrelated with the regressors.

Given a small-enough value of $\beta \geq 0$, one can define the matrix

$$[\mathbf{I} - \beta G]^{-1} = \sum_{k=0}^{+\infty} \beta^p \mathbf{G}^p \tag{8}$$

The $p - th$ power of the matrix G collects the total number of paths, both direct and indirect, in the network starting at node i and ending at node j. The parameter β is a decay factor that scales down the relative weight of longer paths, i.e. paths of length p are weighted by β^p. It turns out that an exact strict upper bound for the scalar β is given by the inverse of the largest eigenvalue of $\mathbf{G}$ (Debreu and Herstein, 1953). Recall from above that this quantity is precisely the epidemic threshold in epidemiological models.

In a row-standardized matrix, such as the one used in social interactions models, the largest eigenvalue is 1. If $|\beta| < 1$ expression (8) is well-defined, that is, the infinite sum converges. The fact that this infinite sum is bounded is precisely the reason that in epidemiological models that epidemic do not occur. If $\beta > 1$, the process is explosive. In a systemic risk context, it is equivalent to a complete financial collapse. While interesting in its own right, for our purposes the more interesting case is how, even in the absence of a complete financial collapse, a small shock can cascade causing large, measurable and quantifiable damage. The matrix $[\mathbf{I}-\beta G]^{-1}$ is able to capture all the effects that stems from a given network topology, that is the cascades of effects stemming from direct and indirect connections.

If one solves for $\boldsymbol{\omega}$ in model (7), the result is a reduced form relationship under the assumption that $|\beta| < 1$:

$$\boldsymbol{\omega} = [\mathbf{I}-\beta G]^{-1} [\gamma\mathbf{I} + \delta G] \, \mathbf{x} + [\mathbf{I}-\beta G]^{-1} \boldsymbol{\epsilon} \tag{9}$$

The primary object of interest is the estimate of the coefficient β. It measures the *average* correlation in outcomes between connected agents. Our approach allows us to understand how this average correlation leads to changes in outcomes for others in the network. For example, given an average β of 90%, what the specific impact of a shock to a given agent and what is the aggregate impact across the network?

We denote by λ the *aggregate* impact. It is the infinite sum of the impacts of a shock across all agents, where each successive link has a smaller impact. We believe this a measure of systemic risk and can be simply computed as

$$\lambda = 1 + \beta + \beta^2 + ... = \frac{1}{1-\beta}.$$

For example, if the outcomes of interest are some returns across traders and we estimate β equal to 0.9, then a change of 5% in the returns of a given trader translate into an average 4.5% for those directly connected. Notice that β is the measure of *average* correlation in returns between trader. If return of trader i changes by 5% and β is 0.9 then connected traders returns change on average 4.5%. The returns of those connected to the connected traders would change on average by 4.5%*.9=4.05%, etc. The limit of this series can be calculated to show that the total impact on the system through all connected traders is $\lambda = \frac{1}{1-0.9} = 10$. This evidence would suggest that shocks can be amplified as much as 10 times. Notice of course that λ is an average measure of risk in the market. Depending on the particular structure of the network, individual traders can pose more or less risk. The extent and speed of the propagation of the shock depends on the network structure G and accrue to each trader according to her position in such structure.

Notice that the above equation (2) is a tool to measure the correlations between the returns of traders that trade with one another in this market. To move beyond the presence of correlations in the data to a statement about the role of network in determining returns requires one to rule out the presence of any factors that are correlated both with network structure and with returns. If traders can choose the traders with whom they trade, then there might be some unobserved factors driving both network formation and outcomes. Our approach to the analysis of systemic risk thus holds true if the network G is exogenous. In other words, this approach is able to quantify how much a shock is amplified by the network and how widely it is transmitted once strategies are realized (i.e. for a given network of realized trades).

In the remainder of this paper we show the ability of this approach to capture patterns in financial data using an application where the network topology can be considered as exogenous.

3 An empirical application: the CME market

We discuss how network effects and systemic risk can emerge in a market with a central counterparty. By providing details on the spread of risk and the sources of profitability at this level of disaggregation will help with an understanding of systemic risk and with the development of policy. We provide evidence of the presence of networks effects in anonymous electronic markets. From this, we discuss how these networks may emerge as a result of particular trading strategies. We do not explicitly model asset prices, but the implication of the presence of strategic behavior in this environment is that decision making may be impacted by the presence of networks, pointing to the need to model strategic behavior on networks.

The Chicago Mercantile Exchange (CME) organizes a wide variety of futures markets. Among the most important in its volume and relevance to other markets is the DOW futures market. This market, as well as the other runs by the CME, are computer intermediated exchanges. Traders submit orders, and a computer match-maker link buyers and sellers according to price and time priority. Traders cannot directly pick their counterparty. It is in this setting that we evaluate the importance of networks in the evaluation of systemic risk.

What do we mean by a network? We will define it simply as the realized pattern of trades. After each transaction is completed, the CME records the buyer, seller, number of contracts, price and time of transaction. This data show which trader bought and which trader sold a given number of contracts at a given price at a particular time of day. To understand the importance of connections, if there is any, we subdivide our data into a sequence of trading periods. Then for each trading period, we can create a mapping of the links that took place during that time. While we don't have strong priors about the correct trading period, we begin with a small time window and define 250 trades as our baseline trading period. We then consider denser networks consisting of 500 and 1000 trades.[5] Figure 1 shows an example of an observed network of 500 trades.

[insert Figure 1 here]

The picture shows a distribution of trading connections which is highly heterogenous, with a trader connected to most of the others and a number of almost isolated traders. The heterogeneity in the distribution of contacts in a population is one of the key factors affecting the propagation of diseases (Pastor-Satorras and Vespignani, 2001). A large variance of the degree (the number of a node neighbors) distribution is a typical feature of complex networks (Albert and Barabási, 2002). In our empirical exercise we aim at capturing the relationship between network structure and profitability.

Because this market is so liquid, traders can buy and sell their positions nearly instantaneously. As a result, we can look at returns by marking to market any transaction that occur during a time period to the end-of-period price. We do so for each trader and each transaction over the time period of our data. The transactions take place during August 2008 during the time when the markets for stocks underlying the indices are open: weekdays between 9:30 am EST and 4:00 pm EST. We have a total of 21 trading days.

With the network topology at hand, we then estimate model (7) and get the estimated correlation in returns between linked traders, β. We aim at answering questions such as: if a trader earns 5% in a given time period, how much does a linked trader earn? How much do the second-degree

[5] These networks are denser because the same number of agents will have more connections, so the matrices shown above would have more '1' values and fewer '0' values.

links earn? As well, we can evaluate the role of networks in propagating changes in returns across traders and obtain a measure of systemic risk in the system.

Our approach here is to describe the strategic interactions at work in this market.

Consider a group of traders. These traders enter each day with a set of trading strategies. These strategies can either be formal or informal, automated or manual. Indeed, the market contains some of each of these. Among these formal strategies, for example, are algorithmic traders. These computerized high-frequency traders composed approximately one-third of volume (Kirilenko et al, 2011) on any given day. The strategy of any given trader will depend on the anticipated strategies of other traders as well as the observed actions during the day. As successful strategies become known, followers emerge and copy the strategy. As long as traders either use correlated strategies or condition their strategies on like information, their behaviors may be correlated in equilibrium and thus as well in the observed data.

Of course, these correlated bidding patterns lead to similarity in returns. Because the matching algorithm used by the CME is blind to identities of the traders, traders with correlated strategies will trade with each other as well as with others. As they do so, and form links with one another, correlation in trading strategies leads to a connection between strategies and network position. Sophisticated traders can then optimize both over strategies, but also potentially over the impact of the resulting network position itself. Many traders will acknowledge that sitting between two traders with fundamental liquidity needs can be profitable. We illustrate this in figure 2.

[insert Figure 2 here]

Note that futures markets are zero-sum markets in aggregate. Thus, while each transaction could potentially yield a profit for both parties, some portion of the network must absorb equal losses for each gain. To see how two traders could both profit from a transaction, consider the presence of two large traders (denoted 'A' & 'D') that have fundamental liquidity demands, one positive and one negative. Each of these participate in the futures market by placing large one-sided orders either to buy or sell contracts.

A separate set of traders, denoted 'B', implements rapid offers to buy and sell. The objective of such traders is to provide the liquidity needed by the large traders with fundamentals demands. Because the large traders may not appear on the market at precisely the same time, the liquidity providers can extract profits from the large traders by being willing to transact when needed. The combination of the liquidity traders' actions can generate a diamond-shaped network pattern illustrated in Figure 2, Panel A. On one side, the liquidity traders buy as needed and on the other they sell as needed. By being willing to buy and sell, the agents in the center can generate profits.

Of course, knowledge that agents can achieve these profits leads to a new set of trading strategies. Figure 2 shows the emergence of additional agents, denoted 'C'. Effectively, the second set of agents hopes to intermediate between one large trader with fundamental demand and the initial set of liquidity traders.

Now, if one evaluates the correlation in returns over a given period of time of these traders, she will observe that the profits of the liquidity traders are inversely correlated with those at the ends of the diamond. As the large traders lose money the liquidity traders earn (or vice-versa). However, our new entrants 'C', over time, yield returns that are positively correlated with the other liquidity traders.

An example is presented in Panel B. The table shows the returns of each set of traders in a hypothetical case. The outcome in case 1 is that the returns of A and D are negatively correlated with the returns of B. This would generate a negative systemic risk coefficient. That is, the market will act like a shock absorber. As new shocks hit the system, the reaction is to ameliorate the impact.

In the second case, the returns of A and D continue to be negatively correlated with B and C; however, B and C show positively correlated returns. It is straightforward to see that as the number of traders in the center of diamond increases, the systemic risk coefficient will become more positive.

To see how two traders could both profit from an interaction, consider A, B, and C. A purchases a contract from C for \$2 at time t and buys one from B for \$1 at time $t+1$. At time $t+2$, C purchases a contract from B for \$1.25. The final transaction yielded C a profit of \$0.75 and B a profit of \$0.25. Of course, A has lost the full dollar in the process. The trade between A and C allows them to share the \$1 gain. Repeated interactions of this type will generate a positive spillover.

These spillovers help understand how returns can be correlated across trading strategies, but more importantly they help illustrate how shocks are propagated. That is, they are a representation of the pathways of systemic risk transmission.

The DOW futures dataset consists of 1,163,274 transactions between approximately 7,335 trading accounts. We compute returns and volumes for each trader and show in Table 1 some sample statistics of the data for each definition of networks. Returns are shown as absolute levels of holding at end of time period based on an initial investment of \$1; thus, a return of 1 indicates that the trader broke even during the time period. Returns shown are period-specific returns, where each period is 250 transactions. Average returns vary from a loss of 4 basis points to a gain of 4 basis points. Of course, individual level results vary more widely. Of note is that the average return across trading accounts is below 1, suggesting that traders with high volume, on average, earn higher returns. Also note that the standard deviation of returns and volume is increasing in the density of the networks. As the number of transactions increase, the variance does so as well.

Notice that the mean transaction volume declines as the number of transactions (i.e. density of network) increases. This pattern reflects the skewness in the data. There are large numbers of traders with low volume and small negative returns, and a relatively smaller number of observations with higher volumes and/or positive returns.

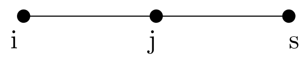
[insert Table 1 here]

The networks of transactions that we define have a couple of important properties. First, they are distinct from one another over time. This occurs both because agents may not be active in each time period and because their transactions are matched by the trading algorithm in each time period. This results in a series of unique networks. Second, the networks are characterized by exogenous link formation. That is, because the links between agents are formed by price and time priority alone, agents cannot effectively choose their partners.

3.1 Estimation results

For each type of network (250, 500 and 1000 trades), we separately estimate our model (7) for each trading day and report a range of estimation results across the observed 21 days. Model (7) is a traditional spatial autoregressive model and can be estimated using Maximum Likelihood using packages implemented in standard software, such as Stata.

Because there is little reason to believe that in an electronically matched market traders' characteristics would generate any troubling effect, we do not consider any x variables. However, the volumes of trades has to be taken into account. We weight the links within the network using the total trading volume in the same trading period of each i and j. For example, in the simple network structure



one could specify the number of contracts sold in each transaction so that the interaction matrix appears as:

$$G = \begin{array}{cccc} & i & j & s \\ i & 0 & 4 & 0 \\ j & 0 & 0 & 2 \\ s & 3 & 0 & 0 \end{array}$$

indicating that i sold j 4 contracts, s sold i 3, and j sold s 2 contracts. Notice that the intransitivities remain and identification is preserved. This weighting has no impact on the techniques discussed here (see Newman, 2004, for a discussion).

Table 6 contains the estimation results, including network fixed effects to control for unobserved heterogeneity. Using the network pattern of trades, our regressions explain more than 70% of the variation in returns. The implication of the very high R squared values is that the network structure is a primary mechanism that determines returns. The estimates of β are highly statistical significant. Their estimated magnitude shows that β is close to one. This indicates that the discount factor in expression (7) remains strong even for traders further away in the network chain of contacts. Therefore, this evidence suggests that the effect of a shock will not be local but it will extend widely across the entire network structure.

Recall that because the networks themselves are exogenous, the relationship that we find between network structure and returns is a *causal* one. Indeed, given the matching process used in this market, the variations in returns can be partially attributed to the network structure itself.

Finally, Table 6 reports the average systemic risk multiplier, λ. We can see that the multiplier is between 5 and 21. These large numbers imply that these trading networks have very high sensitivity to shocks. Small changes to agents on average are magnified up to 21 times the initial shock.

[insert Table 2]

4 Conclusions and policy implications

Much of our motivation for this chapter is the presence of strategic behavior in the financial markets. This strategic behavior can be captured in empirical network models in the way we described. An alternative (although related) approach would be to estimate fully structural models of strategic behavior in financial markets, which would also allow to consider the process of network link formation. Cohen-Cole et al. (2011) develops a static and dynamic microfounded model of bank behavior in the interbank market. In that model, one can draw a direct link between the Nash equilibrium of the model and the measure of the systemic risk, λ, that we discuss here.

The existing studies, however, are only a portion of the needed advances in this literature. The research area on networks and systemic risk continues to face a number of challenges. Among these are the precise identification of systemically important financial institutions (SIFIs), the modeling of the impact of regulatory interventions on market structure, an understanding of how central bank policy impacts market prices and liquidity. To identify SIFIs, the modeling approach in Liu et al (2011) is potentially promising. Using a games on networks approach, it calculates the benefit to a system of removing a key, central player. Using this approach, one could potentially identify which bank's removal from the network causes the most harm. By removing each bank from the network, one can determine how networks reform in each case, and then measure the cost to the system of this disruption.

To understand policy interventions and assess them requires some type of welfare analysis that is precise to the actual network structure of the market and reflects the incentives present in the market. At this stage, the literature offers some descriptive insights but little ability to systemically evaluate policy. A parallel could be drawn here to the literature on monetary policy. For many years, and well into the 20th century, policymakers had only partially formed views of the goals of monetary policy. Policymakers largely leapt from crisis-to-crisis with a general approach, but without fully articulated methods or a clear objective function. Current research has now infused central banking to the extent that a clear policy objective is often directly included in bank mandates.

The approach to the management of systemic risk is currently in that same formative phase. While bankers largely know that they wish to preserve financial stability, they do so mostly based on incomplete, non-structural views of the marketplace and appear to leap from crisis to crisis.

Obviously, one would like for policymakers to define a clear objective function relating to systemic risk. From this, one would want them to have a clear set of rules that emerged from their objective function and a structural view of the economy.

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Table 1: Summary Statistics

Note: Returns are defined as the gross return on an investment; thus a value of 1 indicates no change in value. Values greater than one are net gains and those less than one are net losses. For each density of network, we report the average daily return as well as the total daily volume at the trader level. Thus, we report the mean return across individual level traders, where for each trader, we have calculated their own average return over the course of the trading day. Note that these trader-level returns are unweighted by volume. Because the futures markets are zero-sum, volume weighted returns are zero by construction. Volumes statistics are average daily volumes at the level of the trader. Standard deviations are measured as the variance over the returns at the trader level, again unweighted. Minimums and maximums are the smallest and largest for a trader on any day.

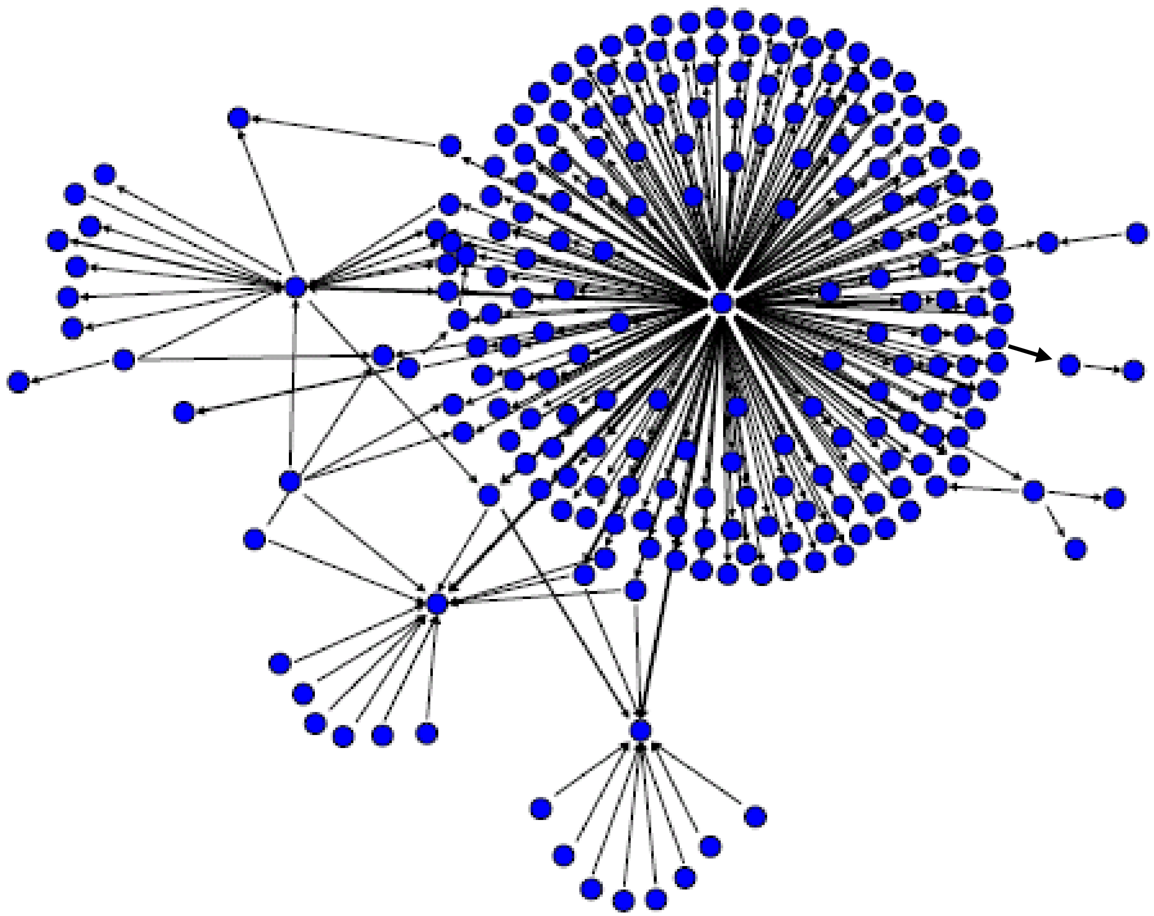
DOW futures	**Mean**	**Standard Deviation**	**Min**	**Max**
Networks of 250 transactions				
Average Returns (unweighted)	0.99	0.03	0.99	1.02
Average Daily Volume	6.39	1.42	1.00	150
Networks of 500 transactions				
Average Returns (unweighted)	0.98	0.05	0.98	1.03
Average Daily Volume	6.33	2.60	1.00	190
Networks of 1000 transactions				
Average Returns (unweighted)	0.95	0.07	0.98	1.04
Average Daily Volume	5.91	4.86	1.00	341
Total number of transactions	1,163,274			
Total number of # trading accounts	7,335			

Table 2: Estimation

Note: The table shows results from the Dow futures market. We perform a maximum likelihood estimation of the spatial autoregressive of individual returns on the network patterns of the returns of the rest of the market (model 7). The adjacency matrix of realized trades is a symmetric, non-directed matrix of 1's and 0's with 1's indicating the presence of a trade and 0 the absence. This table uses a weighted matrix defined as the element-by-element product of the adjancency matrix of realized trades and the sum of trading volume. We thus estimate weighted network effects that reflect the relative importance of traders in the system. We also include network fixed effects to control for unobserved heterogeneity, meaning that we use the deviation in returns from the average return at the network level in each time period. For each network density, we report the range of estimation results across 21 trading days. The first row shows the estimates of the parameter β, the network effect coefficient. T-statistics are reported below coefficient estimates. Below, we include the adjusted R squared value from each specification and the average systemic risk multiplier. This multiplier is total network impact of a one unit shock to an individual. Averaging across the impact for all individuals in the network produces this number, which is equal to 1/(1-β). A constant is included in all regressions. We denote statistical significance of coefficients at the 10, 5 and 1% levels with ***, **, and *, respectively.

	Networks of 250 transactions		**Networks of 500 transactions**		**Networks of 1000 transactions**	
	lowest	***highest***	***lowest***	***highest***	***lowest***	***highest***
DOW futures						
Network Effect Coefficient (β)	0.82***	0.88***	0.84***	0.92***	0.90***	0.95***
t - statistic	385.74	448.79	307.21	411.65	237.46	310.21
Constant	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*
Fixed Effects	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*
R-Squared	0.71	0.78	0.70	0.79	0.71	0.78
Average Systemic Risk Multiplier (λ)	5.56	8.33	6.25	12.98	10.20	19.59

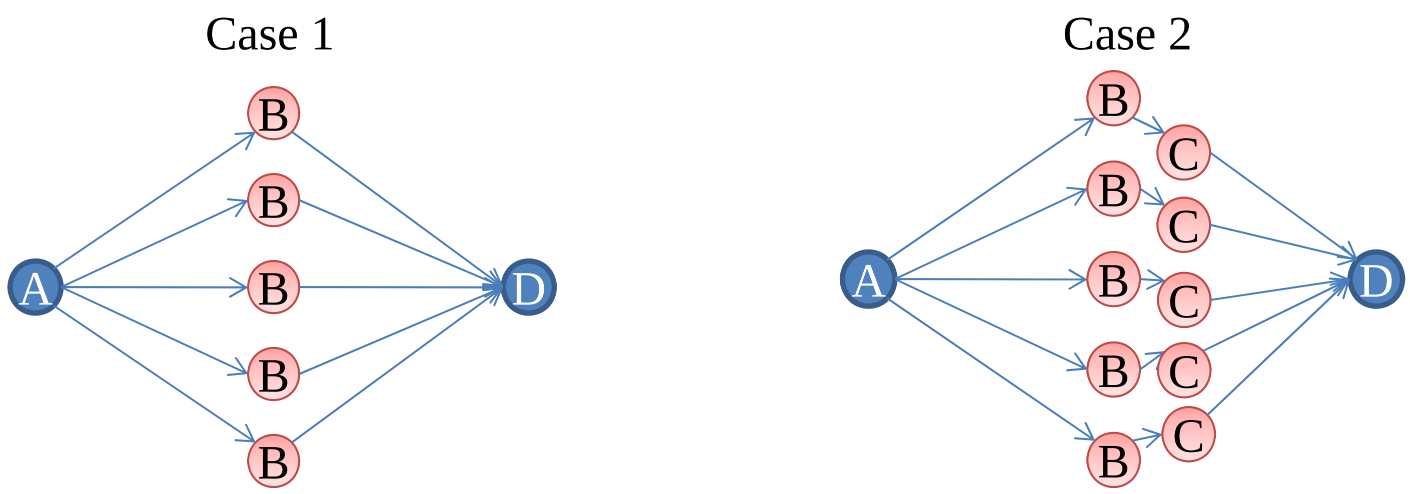
Figure 1



Note: Figure 1 shows one actual network realization of 500 transactions Each arrow shows a single transaction

Figure 2

Panel A



Panel B

	Case 1		Case 2	
	Action	Returns	Action	Returns
A	Sell 100 at 99	0.99	Sell 100 at 99	0.99
B	Buy 100 at 99	1.02	Buy 100 at 99	1.01
	Sell 100 at 101		Sell 100 at 100	
C			Buy 100 at 100	1.01
			Sell 100 at 101	
D	Buy 100 at 101	0.99	Buy 100 at 101	0.99

Note: Panel A shows two agents with fundamental liquidity needs, marked A and B, and a series of agents that have traded with them. Each edge is marked as an arrow, pointing from the seller to the buyer. Panel B shows the same configuration with the addition of a few additional agents. The example assumes that the market price is constant at 100.